GUJARAT UNIVERSITY

B.E. SEM – 3 (MECH/AUTO/MME)

Question Bank

Applied Maths – II

Each question is of equal Marks (10 Marks)

Q.1	Find the Fourier Series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.
Q.2	Expand $f(x)=x\sin x$ as a Fourier series in the interval $0<x<2\pi$.
Q.3	Find the Fourier series of $f(x)=2x-x^2$ in the interval (0,3). Hence deduce that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-...=\frac{\pi^2}{12}$.
Q.4	Find the Fourier series of the function $f(x)=\begin{cases} x^2 & 0\le x\le \pi \\ -x^2 & -\pi\le x\le 0\end{cases}$.
Q.5	Find the Fourier series of the function $f(x)=\begin{cases} \pi x & 0<x<1 \\ 0 & x=1 \\ \pi(x-2) & 1<x<2\end{cases}$. Hence show that $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...=\frac{\pi}{4}$.
Q.6	Find the Fourier series of $f(x)=x^2$ in the interval $0<x<a$, $f(x+a)=f(x)$.
Q.7	If $f(x)=\lvert\cos x\rvert$, expand $f(x)$ as a Fourier series in the interval $(-\pi,\pi)$, $f(x+2\pi)=f(x)$.
Q.8	For the function $f(x)$ defined by $f(x)=\lvert x\rvert$, in the interval $(-\pi,\pi)$. Obtain the Fourier series. Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...=\frac{\pi^2}{8}$.
Q.9	Given $f(x)=\begin{cases} -x+1 & -\pi\le x\le 0 \\ x+1 & 0\le x\le \pi\end{cases}$. Is the function even of odd ? Find the Fourier series for $f(x)$ and deduce the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...$.
Q.10	Find the Fourier series of the periodic function $f(x)$; $f(x)=-k$ when $-\pi<x<0$ and $f(x)=k$ when $0<x<\pi$, and $f(x+2\pi)=f(x)$.
Q.11	Half range sine and cosine series of $f(x)=x(\pi-x)$ in $(0,\pi)$
Q.12	Find the Fourier series for the function $f(x)=\begin{cases} \pi x, 0<x<1 \\ \pi\left(x-2\right), 1<x<2\end{cases}$
Q.13	Find the Fourier series for f(x) defined by f(x) = $x+\frac{x^2}{4}$ when $-\pi< x <\pi$ and f(x + 2π) = f(x) and hence show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.......=\frac{\pi^2}{12}$
Q.14	Find the Fourier series for the function $f(x)=\begin{cases} x; 0<x<1 \\ 0; 1<x<2\end{cases}$.

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Q.15	If f(x) = x in $0 < x < \frac{\pi}{2}$ $= \pi - x$ in $\frac{\pi}{2} < x < \frac{3\pi}{2}$ $= x - 2\pi$ in $\frac{3\pi}{2} < x < 2\pi$ Prove that f(x) = $\frac{4}{\pi}\left\{\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \right\}$
Q.16	If f(x) = $\frac{x}{l}$ when $0 < x < l$ $= \frac{2l - x}{l}$ when $l < x < 2l$ Prove that f(x) $\frac{1}{2} - \frac{4}{\pi^2}\left(\frac{1}{I^2}\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \frac{1}{5^2}\cos\frac{5\pi x}{l} + ...\right)$
Q.17	When x lies between $\pm\pi$ and p is not an integer, prove that sin px = $\frac{2}{\pi}\sin p\pi\left(\frac{\sin x}{1^2 - p^2} - \frac{2\sin 2x}{2^2 - p^2} + \frac{3\sin 3x}{3^2 - p^2} - ....\right)$
Q.18	Find the Fourier series for the function $f(x) = e^{ax}$ in $(-l, l)$
Q.19	Half range sine and cosine series of $f(x) = 2x - 1$ in $(0,1)$
Q.20	Half range sine and cosine series of x^2 in $(0, \pi)$
Q.21	Find Half range sine and cosine series for $f(x) = (x-1)^2$ in $(0,1)$
Q.22	Evaluate: $L\{\sin 2t \cos 3t\}$, $L\{e^{-3t}(\cos 4t + \sin 2t)\}$
Q.23	Evaluate: $L\{\sin^2 2t\}$, $L\{e^{-2t}\cos 3t\}$

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Q.24	Evaluate: $L\left\{\frac{\sin 2t - \sin 3t}{t}\right\}$, $L\left\{t\int_0^t e^{-4t}\sin 3t\,dt\right\}$
Q.25	Evaluate: $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$, $L^{-1}\left\{\frac{s^2+s+2}{s^5}\right\}$
Q.26	Evaluate: $L^{-1}\left\{\cot^{-1}\frac{s}{a}\right\}$, $L^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\}$
Q.27	Evaluate: $L^{-1}\left\{\log\left(\frac{s+2}{s+3}\right)\right\}$, $L^{-1}\left\{\frac{s+2}{\left(s^2+4s+5\right)^2}\right\}$
Q.28	Evaluate: $L^{-1}\left\{\frac{1+2s}{(s+2)^2\,(s-1)^2}\right\}$, $L^{-1}\left\{\frac{s^2+s+3}{s^6}\right\}$
Q.29	Evaluate: $L^{-1}\left\{\frac{(s+1)^2}{s^3}\right\}$, $L^{-1}\left\{\tan^{-1}\frac{s}{a}\right\}$
Q.30	Find the Laplace Transform of f(t), where (i) $f(t) = t$ if $0 < t < \frac{a}{2}$, $f(t+a) = f(t)$ $= a - t$ if $\frac{a}{2} < t < a$
Q.31	Find the Laplace transform of the function $f(t) = \begin{cases} \sin \omega t; 0 < t < \frac{\pi}{\omega} \\ 0; \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$, $f(t) = f(t + \frac{2\pi}{\omega})$
Q.32	Use convolution theorem to find the Laplace Inverse Transform of (i) $\frac{sa}{\left(s^2-a^2\right)^2}$ (ii) $\frac{s-2}{s\,(s-4s-13)}$
Q.33	Use convolution theorem to find the Laplace Inverse Transform of

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	(i) $\frac{s^2}{(s^2+a^2)(s^2-b^2)}$ (ii) $\frac{1}{s^2(s-2)}$
Q.34	Find the value of the integral using Laplace Transform technique. (i) $\int_0^\infty t\, e^{-2t} \cos t\, dt$ (ii) $\int_0^t e^{-t} \frac{\sin t}{t}\, dt$
Q.35	Solve the initial value problem $y''+5y'+2y=e^{-2t}, y(0)=1, y'(0)=1,$ Using Laplace transformation.
Q.36	Solve the following Differential Equations using Laplace Transform technique. $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ with $x=2$ and $\frac{dx}{dt}=-1$ at $t=0$
Q.37	Solve the following Differential Equations using Laplace Transform technique. $\frac{d^2y}{dx^2}+y=1$ with $y(0)=1$ and $y\left[\frac{\pi}{2}\right]=0$
Q.38	Solve the following equations : (a) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$ (b) $(D^2 + D) y = x^2 + 2x + 4$
Q.39	Solve the following equations : (a) $(D^2 + 1) y = x^2 \cos x$ (b) $(D^2+1)y = e^{2x} + \cosh 2x + x^3$
Q.40	Solve the following equations : (a) $(D^4+2D^2+1)y = x^2\cos^2 x$ (b) $(D^2 + 2) y = e^{-2x} + \cos 3x + x^2$
Q.41	Solve the following equations : (a) $(D^2 + 2D+1) y = x e^x \sin x$ (b) $(D^2 - 9) y = e^{3x} \cos 2x$
Q.42	Solve the following equations : (a) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$ (b) $(D^3 + 8) y = x^4 + 2x + 1$
Q.43	Solve the following equations :

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	(a) $(D^2 - 1)\, y = x \sin 3x + \cos x$ (b) $(D^2 - 4D + 4)\, y = 2e^x + \cos 2x + x^3$
Q.44	Solve: $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x).$
Q.45	Solve: $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + y = ((\log x)\sin(\log x) + 1)/x$
Q.46	Solve: $(3x+2)\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$
Q.47	Solve: $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x).$
Q.48	Solve: $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x.$
Q.49	Solve by using method of variation of parameters: $\frac{d^2y}{dx^2} + y = \sec x.$
Q.50	Solve by using method of variation of parameters: $\frac{d^2y}{dx^2} + y = \tan x.$
Q.51	Solve by using method of variation of parameters: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$
Q.52	The charge q on a plate of a condenser C is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E\sin pt$ the circuit is tuned to resonance so that $p^2 = \frac{1}{LC}$ if initially the current i and charge q be zero show that for small value of $\frac{R}{L}$,the current in the circuit at time t is given by $\left(\frac{Et}{2L}\right)\sin pt$.
Q.53	Solve the following simultaneous equations: $\begin{matrix} Dx + y = \sin t \\ Dy + x = \cos t \end{matrix}$; where D = $\frac{d}{dt}$

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	given that when t =0 ,x =1 and y = 0.
Q.54	Solve the following simultaneous equations: $\begin{matrix} Dx+y=e^{t} \\ Dy+x=e^{-t} \end{matrix}$; where D = $\frac{d}{dt}$
Q.55	Form the partial differential equation of following: (a) $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ (b) $z=f(x+ct)+g(x-ct)$
Q.56	Form the partial differential equation of following: (a) $2z=a^2x^2+b^2y^2$ (b) $z=x+y+f(xy)$
Q.57	Form the partial differential equation of following: (a) $z=(x^2+a)(y^2+b)$ (b) F(xy+z^2 , x + y+ z) = 0
Q.58	Solve following partial differential equations : (a) $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$ (b) $x(y-z)p+y(z-x)q=z(x-y)$
Q.59	Solve following partial differential equations : (a) $py+qx=pq$ (b) $z=px+qy+2\sqrt{pq}$
Q.60	Solve following partial differential equations : (a) $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}=\sin x\cos y+xy$ (b) $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}=\cos x\cos 2y$
Q.61	Solve following partial differential equations : (a) $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=e^{x+4y}$ (b) $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=x^3+e^{x+2y}$
Q.62	(a) Solve: $\frac{\partial^2 z}{\partial x\partial y}=e^{-y}\cos x$,given that z = 0 when y = 0 and $\frac{\partial z}{\partial y}=0$ when x = 0

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	(b) Solve: $\frac{\partial^2 z}{\partial x^2} = z$ given that $z = e^y$ and $\frac{\partial z}{\partial x} = e^{-y}$ when $x = 0$
Q.63	Solve: $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$ where z (x , 0) = 8 e^{-5x} using method of separation of variables.
Q.64	Solve: $3\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y}$ = 0, where *z (x , 0) = 4* e^{-x} by using method of separation of variables.
Q.65	Solve: $\frac{\partial z}{\partial x} = 4\frac{\partial z}{\partial y}$ where z*(0 , y) = 8* e^{-3y} using method of separation of variables.
Q.66	Find the series solution of the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$
Q.67	Solve the following equation in power series $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$
Q.68	Solve in series in differential equation $\frac{d^2y}{dx^2} + xy = 0$
Q.69	Solve in series in differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$
Q.70	solve in series the differential equation $\frac{d^2y}{dx^2} + 4y = 0$
Q.71	Attempt the following 1) Derive Cauchy –Riemann equations for complex function w = f(z) in polar form. 2) Define harmonic function, Show that $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and determine its conjugate function.
Q.72	Attempt the following 1) Derive Cauchy-Rieman equation for a complex function $W = f(z)$ In polar form .Hence deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. 2) If $w = u + iv$ represent the complex potential function for an electric field and $u = 3x^2y - y^3$, determine the function v.
Q.73	Attempt the following 1) If f(z) is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$

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	2) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $(i) y = x (ii) y = x^2$
Q.74	Attempt the following 1) Find the image of infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ also show the region graphically. 2) Define line integral .Evaluate $\int_0^{3+i} z^2 dz$ along (i) the line $y = \frac{x}{3}$; (ii) the parabola $x = 3y^2$
Q.75	Attempt the following 1) Define bilinear transformation, Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle onto the straight line $4u + 3 = 0$ 2) State Cauchy integral theorem and Cauchy integral formula. Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where $c : \lvert z \rvert = 3$
Q.76	Attempt the following 1) Determine the analytic function whose real part is $u = e^{-x}(x \sin y - y \cos x)$. 2) Find the Bi-linear transformation, which maps the points $z = -1, i, 1$ into the points $w = 1, i, -1$.
Q.77	Attempt the following 1) Find the Bi-linear transformation which maps the points $z = 1, i, -i$ into the Points $w = 0, 1, \infty$. 2) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where *C* is the circle $\lvert z \rvert = 2$.
Q.78	Attempt the following 1) Prove that $f(z) = e^{2z}$ is analytic every in the plane and find its derivative. 2) Find the image of $\lvert z - 2i \rvert = 2$ under the mapping $w = \frac{1}{z}$
Q.79	Attempt the following 1) Evaluate $\int_{1-i}^{2+3i} (z^2 + z)dz$ along the line joining the points $(1,-1)$ *and* $(2,3)$.

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	2) Evaluate $\oint_C \frac{2z+1}{z^2+z}dz$; where C *is* $\lvert z\rvert=\frac{1}{2}$.
Q.80	Attempt the following 1) If $w=u+iv$ represent the complex potential function for an electric field and $u=3x^2y-y^3$, determine the function v. 2) State the Residue theorem and evaluate $\oint_C \frac{2z+1}{(2z-1)^2}dz$, where C is the circle $\lvert z\rvert=1$.
Q.81	Attempt the following 1) Prove that $f(z)=e^{2z}$ is analytic every in the plane and find its derivative. 2) Expand $f(z)=\frac{z}{(z+1)(z+2)}$ about $z=-2$.
Q.82	Attempt the following 1) Evaluate $\int_0^{3+i} z^2dz$ along (i) the line $y=\frac{x}{3}$ (ii) the parabola $x=3y^2$ 2) Find the image of the upper half plane under the transformation $w=\frac{z}{i-z}$.
Q.83	Attempt the following 1) Find the Bi-linear transformation which maps the points $z=1,i,-i$ into the Points $w=0,1,\infty$. 2) Determine the analytic function whose real part is $y+e^x\cos y$.
Q.84	Attempt the following 1) Evaluate $\int_C \frac{z^2+1}{z(2z+1)}dz$ where C is $\lvert z\rvert=1$. 2) Under the transformation $w=\frac{1}{z}$ find the image of $x^2-y^2=1$.
Q.85	Attempt the following 1) Find the analytic function whose imaginary part is $e^x\sin y$ 2) Under the transformation $w=\frac{1}{z}$ find the image of $\lvert z-2i\rvert=2$.
Q.86	Attempt the following 1) Evaluate $\int_C \frac{z^2+1}{z(2z+1)}dz$ where C is $\lvert z\rvert=1$.

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	2) Under the transformation, $w=\frac{1}{z}$ find the image of $x^2-y^2=1$.
Q.87	Evaluate: (i) $\oint_c \frac{e^{2z}}{(z+1)^4}dz$; where $c:\lvert z\rvert=4$ (ii) $\oint_c \frac{e^{z}}{(z+1)^4(z-2)}dz$; where $c:\lvert z-1\rvert=3$